

# Addition Limits

Tom Kelliher, CS 240

Feb. 24, 2010

## 1 Administrivia

**Announcements**

**Assignment**

**From Last Time**

Decoders, encoders, muxes.

**Outline**

1. A “fast binary adder.”
2. The lower bound for addition speed.

**Coming Up**

Carry lookahead addition, other integer representations.

## 2 A “Fast Binary Adder”

1. Ripple carry adder: example of reuse and divide and conquer.

(a) Wire together  $n$  full binary adders in order to add two  $n$  bit numbers.

(b) Running time of a ripple carry adder.

Running time of a full binary adder is  $O(1)$ . Ripple carry adder?

**Excessive!!!**

2. An  $O(1)$  (!!!)  $n$  bit adder.

(a) Important equations (briefly explain):

Carry generate at bit position  $i$ :  $G_i = A_i B_i$ .

Carry propagate at position  $i$ :  $P_i = A_i \oplus B_i$ .

(b) Carry-in is  $C_0$ .

$$C_1 = G_0 + P_0 C_0.$$

$$C_2 = G_1 + P_1 G_0 + P_1 P_0 C_0.$$

$$C_3 = G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0.$$

Etc.

(c) What's the circuit depth of  $C_i$ ?

(d) What's wrong with this picture?

### 3 The Lower Bound for Addition Speed

1. Due to Winograd.

2. Our abstract gate for an idealized adder: the  $(f, r)$  gate

(a) Independent of technology.

(b)  $f$  is the fan-in.

- (c)  $r$  is the radix (binary).
  - (d) Assume that such a gate can compute any  $r$ -valued function of  $f$  inputs.
3. Adding  $n$  bit numbers. How many inputs? How many outputs? What is the *minimum* number of inputs some output is dependent upon? The *maximum*?
4. For the output dependent upon that maximum, consider the ideal circuit which reduces its inputs to the final output.
- (a) What does it look like?
  - (b) How many intermediate signals remain after one level of gates?
  - (c) What is its depth? — That's the lower bound for addition.