

Rules of Inference, 1.5

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| Modus Ponens | $\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$ |
| Modus Tollens | $\begin{array}{l} \neg q \\ p \rightarrow q \\ \hline \therefore \neg p \end{array}$ |
| Hypothetical Syllogism | $\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$ |
| Disjunctive Syllogism | $\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$ |
| Addition | $\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$ |
| Simplification | $\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$ |
| Conjunction | $\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$ |
| Resolution | $\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$ |

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|----------------------------|--|
| Universal instantiation | $\frac{\forall x P(x)}{\therefore P(c)}$ |
| Universal generalization | $\frac{P(c) \text{ for all } c}{\therefore \forall x P(x)}$ |
| Existential instantiation | $\frac{\exists x P(x)}{\therefore P(c) \text{ for some } c}$ |
| Existential generalization | $\frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$ |

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|-------------------------|---|
| Universal modus ponens | $\frac{\forall x P(x) \rightarrow Q(x) \quad P(a), \text{ where } a \text{ is in the domain}}{\therefore Q(a)}$ |
| Universal modus tollens | $\frac{\forall x P(x) \rightarrow Q(x) \quad \neg Q(a), \text{ where } a \text{ is in the domain}}{\therefore \neg P(a)}$ |

Exercises

pp. 72–74: 1; 3 d, e; 5; 7; **14 c**; **23**; **27**.