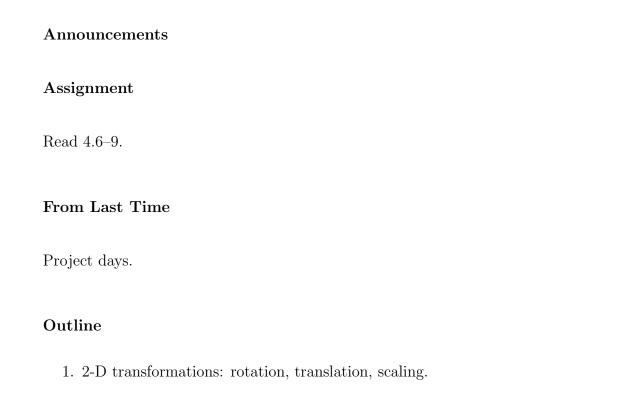
2-D Transformations

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1 Administrivia



Coming Up

Concatenation of transformations, transformation matrices.

2 2-D Transformations

Three primitive transformation:

- 1. Rotation.
- 2. Scaling.
- 3. Translation.

We'll consider each in turn.

The idea is to perform all transformations via matrix multiplications:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} \\ \alpha_{2,1} & \alpha_{2,2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2.1 Preliminaries

For now, we assume you're familiar with:

- 1. Vector spaces and their properties.
- 2. Dot product.
- 3. Magnitude of a vector: $|v| = \sqrt{v \cdot v}$.
- 4. Angle between two vectors:

$$\theta = \cos^{-1}\left(\frac{\upsilon \cdot \omega}{|\upsilon||\omega|}\right)$$

- 5. Properties of matrices.
- 6. Some trigonometry:

$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

$$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

We're all probably somewhat rusty. I know I am.

2.2 Rotation

Consider rotating the point (x, y) by θ about the origin.

$$x = r \cos \phi$$

$$y = r \sin \phi$$

$$x' = r \cos(\theta + \phi)$$

$$y' = r \sin(\theta + \phi)$$

With a little magic:

$$x' = x \cdot \cos \theta - y \cdot \sin \theta$$

 $y' = x \cdot \sin \theta + y \cdot \cos \theta$

What's our transformation matrix look like?

2.3 Scaling

- 1. "Contract" or "expand" a point (polygon).
- 2. Point moves in relation to origin.
- 3. Differential, uniform scalings.

$$x' = s_x \cdot x$$
$$y' = s_y \cdot y$$

Matrix representation?

2.4 Translation

Move the point:

$$x' = x + d_x$$
$$y' = y + d_y$$

Matrix representation?

2.5 Homogeneous Coordinates

- 1. Use allows use to achieve translations via matrix multiplications.
- 2. Add a third coordinate to a point: (x, y, W).
- 3. Two sets of homogeneous coordinates represent the same point iff they are multiples of each other.
- 4. A "homogenized" point.

Our translation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2.6 Composing Transformations

Can we combine transformations?

- 1. Consider composing two translations: d_{x_1} , d_{y_1} and d_{x_2} , d_{y_2} .
- 2. Consider two scalings.
- 3. Consider two rotations.

2.7 Types of Transformations

- 1. Rigid body. Arbitrary sequence of translations and rotations.
- 2. Affine. Parallelism of lines preserved, but not lengths nor angles.
- 3. Shear (affine).

Consider the x-shear transformation:

$$\left[\begin{array}{ccc} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

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What's the y-shear transformation matrix look like?

2.8 General Compositions

- 1. How do we rotate about an arbitrary point?
- 2. How do we scale about an arbitrary point?