Multiplication

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1 Administrivia

Announcements

Homework due Monday.

Exam I in one week, covering all material through Section 4.5 (excepting carry lookahead addition). Review on Wednesday. E-mail me one question by Wednesday, 8:00 AM.

Assignment

Read 5.1–2.

From Last Time

Carry lookahead addition.

Outline

1. Paper and pencil multiplication.
2. Shift and add multipliers.
3. Booth recoding.

Coming Up

MIPS datapath.

2 Multiplication

1. Consider paper and pencil binary unsigned multiplication:

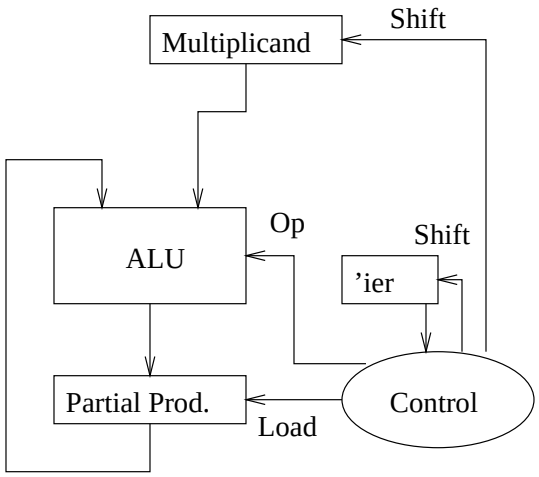
   (a) Shift multiplicand left one bit position after each add/hold cycle.

   (b) Add/hold depending upon current multiplier bit (examine lsb; shift right).

   (c) Example: 1101×0110.

   (d) Multiplying two n-bit numbers results in a $2n$-bit result.

2. Naive 32-bit shift-and-multiply hardware:



   How many bits in the datapath?

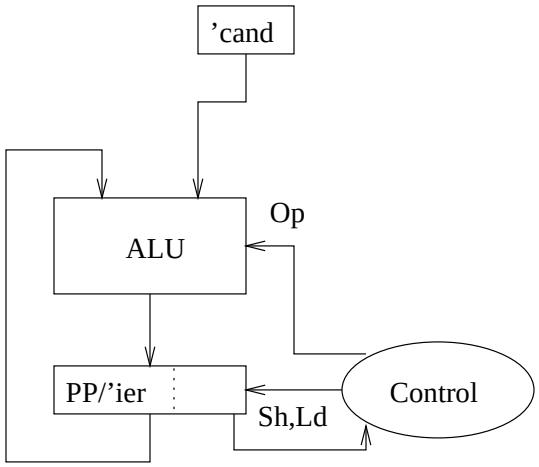
   Running time?

3. Observation: Shifting multiplicand left and keeping partial product stationary is equivalent to keeping multiplicand stationary and shifting partial product right.

   (a) Bonus 1: only 32-bits are added at any one time.

(b) Bonus 2: the multiplier can be stored in the unused part of the partial product register.

More sophisticated 32-bit shift-and-multiply hardware:



4. Optimal running time: $O(\log n)$, achieved with tree of carry-save adders with CLA adder at root.

2.1 Booth Recoding

1. We can speed up multiplication by a factor of two by taking two multiplier bits at a time.

   Problem: must be able to form $3 \times$ multiplicand!

2. Solution:

   (a) Adopt a signed digit set: $\overline{1}, 0, 1$.

   (b) The multiply unit's ALU lets us add *or* subtract.

   (c) Observe that a run of 1's:

   $$0001111100$$

   can be re-written as

   $$0010000\overline{1}00.$$

   Why? All runs of 1's are recoded this way.

(d) Advantage of recoding: now multiplicand need only be multiplied by 1, -1, 2, and -2 — all easy to do with shift and add/subtract.

(e) Example multiply: $29 \times 27 = 783$ or $011101 \times 011011 = 001100001111$. ($-29 = 100011$). Recoded multiplier?

Perform in three cycles. Use 8-bit arithmetic and sign-extend.