APPENDIX A

Math Self-Test

This self-test is designed to help you assess your math readiness. The essentials in order to study computer graphics are coordinate geometry, trigonometry, linear algebra and calculus, all at an elementary level.

Try to answer all 30 questions. Time is not an issue. And feel free to dust off old math books you may have stashed away in some corner, stroll over to the school or public library, or, even, check the internet. The principle is that each and every one of these activities will be allowed throughout your career as a student and practitioner of CG (except, maybe, when you are actually in an exam). Having just the right formula or solution method pop off the top of your head is fantastic, but it's fine as well, given a problem, that you know how to *go about* solving it.

Give yourself 4 points for each correct answer (solutions follow in the next section). If you score at least 100, come on in, the water's fine.* If you're between 80 and 100 then the questions you missed tell where the rust is and, as long as you are willing to put in the extra work, you should be okay. If your score is less than 80 then you need to sit down with yourself and be perfectly honest: is it simply rust that will come off or things that I've just never had in school but trust myself to be able to pick up or is this the kind of stuff that makes me want to curl into a fetal position?

A word about math and CG, especially to those who did not fare well in the test. If you are motivated to study CG then picking up the math on the way isn't just possible, it can be a lot of fun. Its application to CG will bring to life stuff that caused your eyes to roll in high school. "The middle of the spacecraft is light because of the interpolated color values from the ends of the long triangle" or "This matrix will skew the evil character's head" are a lot different from "Groan, that's 12 different theorems and a chapter-load of trig formulas I have to cram for the mid-term."

If you are interested, there are several books out there dedicated to teaching the math needed for CG. A few that come to mind are Dunn [37], Lengyel [87], Mortenson [100] and Vince [151].

*There's more math you'll learn while studying CG (some from this book itself) than is covered in the test. Doing well here simply means you're unlikely to have serious problems.

Appendix A
MATH SELF-TEST

Use the following if you need to (some are approximations): $\sin 30° = \cos 60° = 0.5$, $\sin 45° = \cos 45° = 0.707$, $\sin 60° = \cos 30° = 0.866$, $\pi = 3.141$, $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$.

The first seven questions refer to Figure A.1.

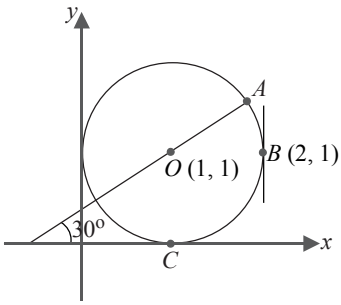

Figure A.1: Circle of unit radius.

1. What is the equation of the circle?

2. What are the coordinates of point C?

3. What is the equation of the tangent to the circle at B?

4. What is the length of the short arc of the circle from A to B?

5. What are the coordinates of point A?

6. If the circle is moved (without turning) so that its center lies at $(-3, -4)$, where then does point B lie?

7. Suppose another circle is drawn with center at A and passing through O. The two circles intersect in two points. What angles do their tangents make at these points (which, of course, is the same as the angles the circles make with each other)?

8. If a straight line on a plane passes through the points $(3, 1)$ and $(5, 2)$, which, if any, of the following two points does it pass through as well: $(9, 4)$ and $(12, 6)$?

9. What are the coordinates of the midpoint of the straight line segment joining the points $(3, 5)$ and $(4, 7)$?

10. At what point do the straight lines $3x+4y-6=0$ and $4x+7y-8=0$ intersect?

11. What is the equation of the straight line through the point $(3,0)$ that is parallel to the straight line $3x-4y-6=0$?

12. What is the equation of the straight line through the point $(3,0)$ that is perpendicular to the straight line $3x-4y-6=0$?

13. What are the coordinates of the point that is the reflection across the line $y=x$ of the point $(3,1)$?

14. What is the length of the straight line segment on the plane joining the origin $(0,0)$ to the point $(3,4)$? In 3-space (xyz-space) what is the length of the straight line segment joining the points $(1,2,3)$ and $(4,6,8)$?

15. Determine the value of $\sin 75°$ using only the trigonometric values given at the top of the test (in other words, don't use your calculator to do anything other than arithmetic operations).

16. What is the dot product (or, scalar product, same thing) of the two vectors u and v in 3-space, where u starts at the origin and ends at $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$ and v starts at the origin and ends at $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \sqrt{3})$, i.e., $u = \frac{1}{\sqrt{2}}\mathbf{i}+\frac{1}{\sqrt{2}}\mathbf{j}$ and $v = \frac{1}{\sqrt{2}}\mathbf{i}+\frac{1}{\sqrt{2}}\mathbf{j}+\sqrt{3}\mathbf{k}$. Use the dot product to calculate the angle between u and v.

17. Determine a vector that is perpendicular to *both* the vectors u and v of the preceding question.

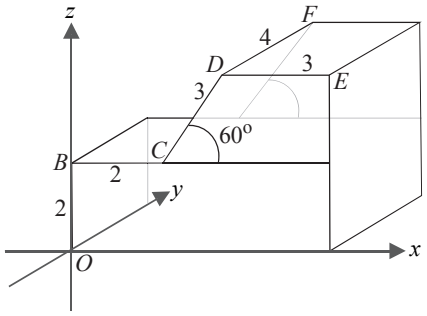

Figure A.2: Solid block (some edges are labeled with their length).

18. For the block in Figure A.2, what are the coordinates of the corner point F?

19. For the block again, what is the angle CDE?

20. For the unit sphere (i.e., of radius 1) centered at the origin, depicted in Figure A.3, the equator (0° latitude) is the great circle cut by the xy-plane, while 0° longitude is that half of the great circle cut by the xz-plane where x-values are non-negative.

    What are the xyz coordinates of the point P whose latitude and longitude are both 45°?

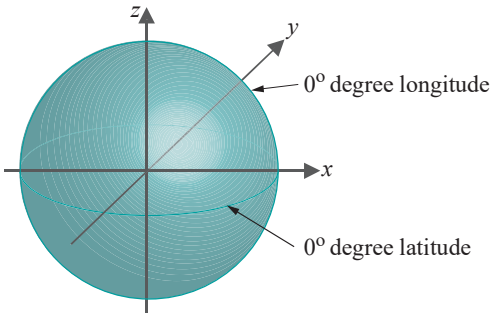

Figure A.3: Unit sphere.

21. Multiply two matrices:

$$\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$$

22. Calculate the value of the following two determinants:

$$\begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} -1 & 2 & -3 \\ 0 & 5 & -2 \\ 0 & 3 & 3 \end{vmatrix}$$

23. Calculate the inverse of the following matrix:

$$\begin{bmatrix} 4 & 7 \\ 2 & 4 \end{bmatrix}$$

24. If the Dow Jones Industrial Average were a straight-line (or, linear, same thing) function of time and if its value on January 1, 2007 is 12,000 and on January 1, 2009 it's 13,500, what is the value on January 1, 2010?

25. Are the following vectors linearly independent?

$$[2\ 3\ 0]^T \qquad [3\ 7\ -1]^T \qquad [1\ -6\ 3]^T$$

26. Determine the linear transformation of $\mathbb{R}^3$ that maps the standard basis vectors

$$[1\ 0\ 0]^T \qquad [0\ 1\ 0]^T \qquad [0\ 0\ 1]^T$$

to the respective vectors

$$[-1\ \ -1\ 1]^T \qquad [-2\ 3\ 2]^T \qquad [-3\ 1\ \ -2]^T$$

27. What is the equation of the normal to the parabola

$$y = 2x^2 + 3$$

at the point $(2, 11)$?

28. If x and y are related by the equation

$$xy + x + y = 1$$

find a formula for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

29. The formula for the height at time t of a projectile shot vertically upward from the ground with initial velocity u is

$$h = ut - \frac{1}{2}gt^2$$

assuming only the action of gravitational acceleration g (ignoring wind resistance and other factors). What is the velocity of the projectile at time t? What is the maximum height reached by the projectile?

30. At what points do the curves $y = \sin x$ and $y = \cos x$ meet for values of x between 0 and 2π? What angles do the curves make with each other at these points?