Question Set 2

CS 420

Chapter 2

1. Identify the types (basis, coordinate vector, matrix, point, vector) of the variables in this equation
$$\vec{v} = \vec{\mathbf{b}}^t M^{-1}\mathbf{c}$$

2. Draw a figure corresponding to
$$\vec{\mathbf{b}}^t\mathbf{c} \Rightarrow \vec{\mathbf{b}}^t M\mathbf{c}$$
and express this mathematical statement in words.

3. Draw a figure corresponding to
$$\vec{\mathbf{b}}^t \Rightarrow \vec{\mathbf{b}}^t M$$
and express this mathematical statement in words.

4. If
$$\vec{v} \cdot \vec{w} = 0$$
then what do we know? Draw a figure corresponding to this.

5. Define the term *3D orthonormal basis*.

6. How is the vector
$$\vec{v} \times \vec{w}$$
related to the two vectors in the expression? Draw a figure corresponding to this.

7. Which of the following are valid expressions in our notation and, if valid, what is the resulting type (invalid, basis, coordinate vector, matrix, point, vector)
   (a) $\vec{\mathbf{b}}^t M$
   (b) $\mathbf{c}M$
   (c) $M^{-1}\mathbf{c}$
   (d) $\vec{\mathbf{b}}^t N M^{-1}\mathbf{c}$

8. Given that $\vec{\mathbf{a}}^t = \vec{\mathbf{b}}^t M$, what are the coordinates of the vector $\vec{\mathbf{b}}^t N\mathbf{c}$ with respect to the basis $\vec{\mathbf{a}}^t$? (Your answer will be a mathematical expression.)

9. Given that the transformation $\mathcal{T}(\vec{v})$ is defined as $\mathcal{T}(\vec{v}) = \vec{v} + \vec{k}$, where $\vec{k}$ is a non-zero constant vector, show that $\mathcal{T}(\vec{v})$ is not a linear transformation, by showing that one of the two properties of linear transformations doesn't hold:
$$\begin{aligned} \mathcal{L}(\vec{u} + \vec{v}) &= \mathcal{L}(\vec{u}) + \mathcal{L}(\vec{v}) \\ \mathcal{L}(\alpha\vec{u}) &= \alpha\mathcal{L}(\vec{u}) \end{aligned}$$