Solving a Recurrence Relation

Use repeated substitution to identify a pattern:

$T(1) = a$

$T(n) = T(n-1) + b$ $\quad\quad T(n-1) = T(n-2) + b$

$= [T(n-2) + b] + b$

$= T(n-2) + 2b$ $\quad\quad T(n-2) = T(n-3) + b$

$= [T(n-3) + b] + 2b$

$= T(n-3) + 3b$

$= T(n-i) + ib$

use the base case to eliminate $T(\sim)$

Base case is $k=1$, so solve

$1 = n - i$ for i $\quad\quad i = n-1$

So $T(n) = T(n-(n-1)) + (n-1)b$

$= T(1) + (n-1)b$

$= a + (n-1)b$

Check using induction

Base step (for the base case)

show $T(1) = a$

$T(1) = a + (1-1)b = a$

Inductive Step

Assume $T(K) = a + (K-1)b$
Show $T(K+1) = a + ((K+1)-1)b$ $= a + Kb$ See note below

$T(K+1) = T(K) + b$ by defn. of the recurrence

$= [a + (K-1)b] + b$ by inductive hypothesis

$= a + Kb$

$\therefore\ T(n) = a + (n-1)b$

$T(n)$ is $O(n)$

Note

If the recurrence equation is of the form

$T(n) = aT(n/b) + \cdots$

Than it's easier in the inductive to show

$T(K) = \cdots$

assuming that your solution is true for all $n < K$.