Show $T(n) = 2^n + 5n$ is $O(2^n)$

I choose $c = 2$ and $n_0 = 5$

So, show $2^n + 5n \leq 2 \cdot 2^n$

for all $n \geq 5$

Basis Step

$n = 5$

Show $2^5 + 5 \cdot 5 \leq 2 \cdot 2^5$

$2^5 + 5 \cdot 5 = 57 \leq 64 = 2 \cdot 2^5$

Inductive Step

Assume $2^K + 5K \leq 2 \cdot 2^K$

for $K \geq 5$

show $2^{K+1} + 5(K+1) \leq 2 \cdot 2^{K+1}$

$$\begin{aligned}
2^{K+1} + 5(K+1) &= 2^K + 2^K + 5K + 5 \\
&= 2^K + 5K + 2^K + 5 \\
&\leq 2 \cdot 2^K + 2^K + 5 \quad \text{(Inductive Hyp)} \\
&\leq 2 \cdot 2^K + 2^K + 2^K \quad (K \geq 5) \\
&= 2 \cdot 2^K + 2 \cdot 2^K \\
&= 2 \cdot 2^{K+1}
\end{aligned}$$

$\therefore 2^{K+1} + 5(K+1) \leq 2 \cdot 2^{K+1}$

Q. E. D.