

# Question Set 6

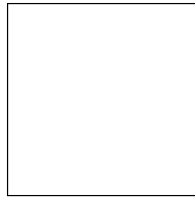
CS 320

## Chapter 7

1. The linear interpolation (LERP) of two rotation matrices,

$$R_\alpha := (R_1 R_0^{-1})^\alpha R_0,$$

when applied to a frame, causes the frame's basis vectors to move along a straight line. This implies that points associated with the frame move along a straight line as well. Consider LERPing two rotation matrices to rotate this square by  $\frac{\pi}{2}$ :



In a single hand-drawn figure, show the result of the LERP operation on this square for  $\alpha = 0$  and  $\alpha = 0.5$ . Explain why this is not rotation.

2. Define axis/angle rotation and explain how a quaternion encodes this rotation mechanism.
3. Show how to construct a quaternion representing a rotation of  $\theta$  about  $\hat{\mathbf{k}}$ . Do not assume that  $\hat{\mathbf{k}}$  is of unit length.
4. What is the multiplicative inverse of the quaternion you constructed in the previous question? Give a geometric reason as to why this is the multiplicative inverse.
5. Assuming that  $\hat{\mathbf{k}}$  is of unit length, show that any quaternion constructed from it is also of unit length.
6. Does the coordinate four-vector  $[\hat{\mathbf{c}}, 1]^t$  represent a point or a vector?
7. Write the rotation by  $\frac{2\pi}{3}$  about  $[3, 1, 5]$  as a unit norm quaternion. Show work.
8. Extract the angle and axis from the quaternion  $[0.8660, 0, 0.2236, 0.4472]$ . Assume that the angle is measured in radians. Show work.