**CS350 – Context Free Grammars and Pushdown Automata**

**Purpose:** The syntax of programming languages can be expressed with context free grammars so they are central to compilers. Pushdown automata are a machine-like view which are equivalent in computation to cfg's. The purpose of this module is for you to express languages using these models and to begin to examine their properties.

**Knowledge:** This module will help you become familiar with the following content knowledge:

- How to express languages using both cfg and pda models.

**Activity:** With your group perform the following tasks and answer the questions. You will be reporting your answers back to the class in 60 minutes.

1. A context free grammar consists of rules involving terminals which are symbols in the alphabet, and non-terminals which can be replaced by the right hand side of a rule. Consider the following cfg with one non-terminal S. S can be replaced by either aSb or by the empty string which we are now denoting with $\lambda$.

   S $\rightarrow$ aSb
   S $\rightarrow$ $\lambda$

   Complete a derivation of the string aabb using this grammar. You can also draw a derivation tree with branches for each symbol on the rhs of a rule.

   S $\Rightarrow$ aSb $\Rightarrow$ ________________$\Rightarrow$ aabb

2. What language $L(G)$ is generated by the grammar $G$ starting with the symbol S. Note that we are using notation which denotes choice in the rhs for A.

   S $\rightarrow$ Ab
   A $\rightarrow$ aAb | $\lambda$

3. The following grammar derives the language that contains all string with an equal number of a's and b's where the symbols can appear in any order.

   $S \rightarrow SS$
   $S \rightarrow aSb$
   $S \rightarrow bSa$
   $S \rightarrow \lambda$

   How would you show that all strings derived by this grammar have an equal number of a's and b's?

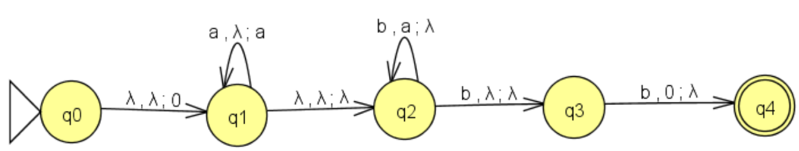
   To show that **all** strings with an equal number of a's and b's are generated takes an induction proof where we induct on the length of the strings. Consider strings with an equal number of a's and b's and start with a and ends with b. Why will all strings of that form be generated?

   Why will all strings with an equal number of a's and b's that start with b and ends with a be generated?

   If a string in the language both starts and ends with a, then it looks like axa where x is a string of a's and b's. Imagine that you march through the entire string axa and add 1 to a count every time you see a and subtract 1 from the count every time we see b. We know that the count must be 0 at the end since we have an equal number of a's and b's. Why must there be some point inside x where the count is also 0? Why would this show that we can generate this string with the first rule of the grammar?

   How would we show that strings that have an equal number of a's and b's and start and end with b are generated?

4. A pushdown automaton (pda) is like a finite automaton that includes a stack. Every transition includes the input symbol being processed; the value that must be at the top of the stack which will be popped off; and the symbol being pushed onto the stack after the previous pop. Consider the following pda and trace through it with the input "aabbbb". Determine what the stack looks like at each step.




   What language does this pda accept?

5. It turns out that with a bit of work that the languages accepted by pushdown automata are equivalent to the languages that can be derived by context free grammars. Check out the appendix in the text for details.

Complete the following assignments for grading. Each should be done individually but you may consult with a classmate to discuss strategies.

**Assignment 1**:
In JFLAP, create and test grammars for the following languages over the alphabet $\Sigma = \{a, b, c\}$ :

1. All strings with exactly two a's.
2. All strings with no more than two a's.
3. $L = \{a^n b^{2n} \mid n \geq 0\}$
4. $L = \{a^n b^m c^{n+m} \mid n \geq 0\}$

**Criteria for Success:** You have thoroughly tested each grammars with strings both in and not in the language and gotten the correct results.

**Assignment 2**:
In JFLAP, create and test pushdown automata for the following languages over the alphabet $\Sigma = \{a, b, c\}$ :

1. $L = \{a^n b^m c^{n+m} \mid n \geq 0\}$
2. $L = \{w \mid \text{number of a's in w} < \text{number of b's in w}\}$

**Criteria for Success:** You have thoroughly tested each pda with strings both in and not in the language and gotten the correct results.

Submit your JFLAP files in Canvas for grading.