CS350 – NP-Completeness

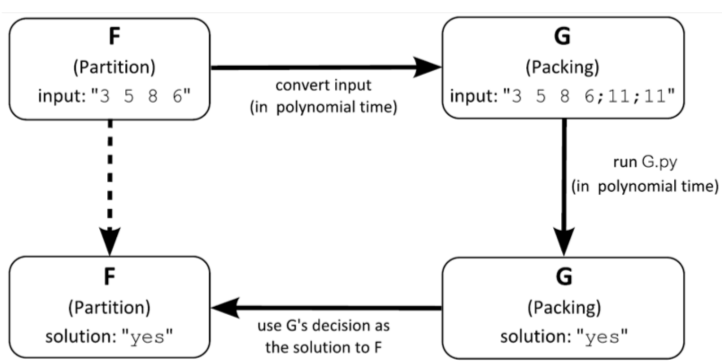
Purpose: The question of whether P is equal to NP is the most famous open problem in computer science. If the answer is yes then many problems that are now difficult to solve will become tractable and this would be a really big deal! One way to do this is to show that any one of the known NP-complete problems, or the hardest problems in the class NP, are solvable in P. The tool for showing a problem is NP-complete is the polynomial time reduction. The purpose of this module is to practice polynomial time reductions and explore the significance of this open problem.

Knowledge: This module will help you become familiar with the following content knowledge:

- How to perform a polynomial time reduction.
- Explore the significance of NP-complete problems.

Activity: With your group perform the following tasks and answer the questions. You will be reporting your answers back to the class in 60 minutes.

1. . A polynomial reduction must have the following properties:
   (a) The reduction maps positive instance to positive instances
   (b) The reduction maps negative instances to negative instance
   (c) The reduction runs in polynomial time



If G is computable in polynomial time, what does that tell us about the computational complexity of F?
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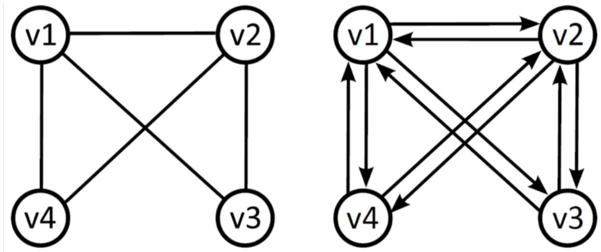
2. Consider the reduction of finding a hamiltonian cycle in an undirected graph and reducing that to finding a hamiltonian cycle in a directed graph. We create an instance of the directed graph by taking each undirected edge and turning it into two directed edges:



G, the original input to Uhc

G', the transformed input to Dhc

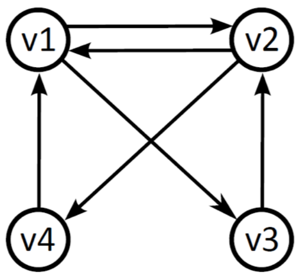
Show that each of the three conditions for a polynomial reduction hold:

Why would a graph with an undirected hamiltonian cycle mean that the directed graph also has a hamiltonian cycle?

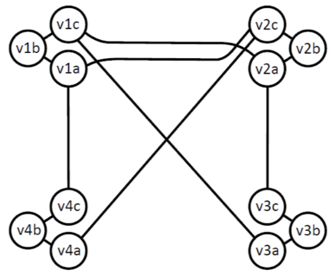
Why would a hamiltonian cycle in the directed graph mean that we also have a hamiltonian cycle in the undirected graph? (This is the contrapositive of the second condition and is an easier way of showing that negative instances map to negative instances).

Why is this conversion from undirected to directed done in polynomial time?

3. We can also reduce the problems from directed to undirected. The key trick is to create an undirected graph where each vertex is now connected triples of vertices:



G, the original input to DHC



G', the transformed input to UHC

Show that each of the three conditions for a polynomial reduction hold:

Why would a graph with a directed hamiltonian cycle mean that the undirected graph also has a hamiltonian cycle?

Why would a hamiltonian cycle in the undirected graph mean that we also have a hamiltonian cycle in the directed graph?

Why is this conversion from undirected to directed done in polynomial time?

4. NP-complete problems are the "hardest" problems in the class NP. We define NP-complete:
Definition: Let C be a problem in NP. We say that C is NP-complete if for all D in NP, D $\leq_p$ C.

What if we are able to show that a particular NP-complete problem is in the class P. What does this tell us about the relationship of the classes P and NP?

5. The problem UHC is NP-complete. This means that all NP problems polyreduce to DHC. Strange but true!

If we already know that a problem C is NP-complete, we can show that a problem D is also NP-complete by doing a polynomial reduction C $\leq_p$ D. Why is that so?

If we already know that UHC is NP-complete, give a polynomial reduction to show that Half-UHC is also NP-complete. A graph has a half-hamiltonian cycle if there is a cycle that visits half the vertices.

Complete the following assignments for grading. Each should be done individually but you may consult with a classmate to discuss strategies.

Assignment 1:
Complete exercise 13.7 on p291 of your text.

Criteria for Success: You show how to take an instance of the stranded salesperson decision problem and convert it to an instance of the traveling salesperson decision problem in such a way that the solution to TSPD will give you a solution to SSPD. The instance of each problem is a weighted undirected graph and a threshold value L and answers "yes" if there is a path of length at most L. You need to take the graph G and threshold L for the SSPD problem and convert it to a G' and L' for the TSPD problem.
You need to show that a) a positive instance of SSPD gives a positive instance of TSPD, b) a negative instance of SSPD gives a negative instance of TSPD (show the contrapositive), and c) the transformation can be done in polynomial time.

Assignment 2:
Complete 13.15 on p292 of your text.

Criteria for Success: You show how to take an instance of the RedDHC problem and turn it into an instance of DHC. As in assignment1 you need to show a) a positive instance of RedDHC gives a positive instance of DHC, b) a negative instance of RedDHC gives a negative instance of DHC (show the contrapositive), and c) the transformation can be done in polynomial time.

Assignment 3:
Complete 14.5 on p311 of your text.

Criteria for Success: You explain what class CRACKRSA is in and how we know.

Assignment 4:
Complete 14.7 on p312 of your text.

Criteria for Success: You describe and cite an NP-complete problem that is substantially different from those we have seen.

Submit your work in Canvas for grading.