1 Administrivia

Announcements

Assignment

Read 3.5. Online quiz.

From Last Time

Recursive functions.

2 A Program for Recursive Systems

See handout.

3 Introduction

1. Geometric growth: Value at time $n$ is directly proportional to value at time $n - 1$.

\[ t_n = kt_{n-1} \]
2. Defn. of geometric progression:

*A sequence (as 1, 1/2, 1/4) in which the ratio of a term to its predecessor is always the same (hence, a constant).*

Compare arithmetic progression.

### 3.1 Examples and Closed Form

1. Blue jeans fade when they are washed: they lost 2% of their color. How much of the original color is left after 50 washes?

Use 1.0.

Growth or decay?

2. Electrical power consumption increases at a rate of 5% per year. This year 500 MW were used. How long before consumption doubles?

Growth or decay?

#### 3.1.1 Closed Form Models

1. Consider the blue jeans example:

\[
\begin{align*}
D_0 &= 1.0 \\
D_1 &= 0.98 \times 1.0 = 0.98 \\
D_2 &= 0.98 \times 0.98 = 0.98^2 \\
D_3 &= 0.98 \times 0.98^2 = 0.98^3 \\
D_4 &= 0.98^4 \\
D_n &= ???
\end{align*}
\]

So: \( y = f(n) = 0.98^n \).

2. Power consumption model. What’s the closed form?

\( f(x) = 1.05^x \times 500. \)

Can we determine more precisely when the consumption doubles? \( x = 14.2067 \) years. Graph \( y = 1.05^x \) and \( y = 2 \) and find the intersection.
Another explicit model: Power usage doubles in 14.2 years. Continuous growth. General model is \( f(t) = 500 \times 2^kt \), where \( k \) is a constant and \( t \) is time in years. Knowing that power usage doubles in 14.2 years, solve for \( k \):

\[
1000 = 500 \times 2^{14.2k}
\]

Solution:

\[
f(t) = 500 \times 2^{\frac{t}{14.2}}
\]

### 3.2 Class Exercise