

# Binary Logic, Gates, and Boolean Algebra

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## 1 Administrivia

**Announcements**

**Assignment**

Read 2.3–4.

**From Last Time**

Introduction

**Outline**

1. Binary logic and gates.
2. Boolean Algebra.

**Coming Up**

Standard forms, maps, and minimization.

## 2 Binary Logic and Gates

1. Fundamental operators and their symbols:
  - (a) AND
  - (b) OR
  - (c) NOT
2. NAND is *complete*.
3. Gate fan-in and fan-out. Electrical significance.
4. Timing diagram.
  - (a) Frequency and period.
  - (b) Timing diagrams. Show AND, OR, NOT waveforms for input: A: 0011, B: 0101.
  - (c) What do the waveforms really look like: propagation delay, noise, under- and over-shoot.

## 3 Boolean Algebra

1. Boolean functions can be represented by equations, truth tables, or logic circuits.

How do you convert from one form to another?

How many rows in the truth table of an  $n$ -input Boolean function?
2. Why would we want to simplify a Boolean equation?
3. Basic Identities:

1. $X + 0 = X$	2. $X \cdot 1 = X$
3. $X + 1 = 1$	4. $X \cdot 0 = 0$
5. $X + X = X$	6. $X \cdot X = X$
7. $X + \overline{X} = 1$	8. $X \cdot \overline{X} = 0$
9. $\overline{\overline{X}} = X$	
10. $X + Y = Y + X$	11. $XY = YX$
12. $X + (Y + Z) = (X + Y) + Z$	13. $X(YZ) = (XY)Z$
14. $X(Y + Z) = XY + XZ$	15. $X + YZ = (X + Y)(X + Z)$
16. $\overline{X + Y} = \overline{X} \cdot \overline{Y}$	17. $\overline{X \cdot Y} = \overline{X} + \overline{Y}$

## 4 Exercises

Example simplifications. Use both a truth table and Boolean manipulation to show:

- $(X + \overline{Y})Y = XY$
- $Y + \overline{X}Z + X\overline{Y} = X + Y + Z$
- $(X + Y)(\overline{X} + Z) = XZ + \overline{X}Y$