2-D Transformations

Tom Kelliher, CS 320

Apr. 1, 2009

1 Administrivia

Announcements

Assignment

Read 4.6–9.

From Last Time

Animation.

Outline

1. 2-D transformations: rotation, translation, scaling.

Coming Up

Project day.
2 2-D Transformations

Three primitive transformations:

1. Rotation.
2. Scaling.
3. Translation.

We’ll consider each in turn.

The idea is to perform all transformations via matrix multiplications:

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} = \begin{bmatrix}
  \alpha_{1,1} & \alpha_{1,2} \\
  \alpha_{2,1} & \alpha_{2,2}
\end{bmatrix} \begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]

2.1 Preliminaries

For now, we assume you’re familiar with:

1. Vector spaces and their properties.
2. Dot product.
3. Magnitude of a vector: \(|v| = \sqrt{v \cdot v}\).
4. Angle between two vectors:
   \[
   \theta = \cos^{-1}\left(\frac{v \cdot \omega}{|v||\omega|}\right)
   \]
5. Properties of matrices.
6. Some trigonometry:
   \[
   \begin{align*}
   \cos(\theta + \phi) &= \cos \theta \cos \phi - \sin \theta \sin \phi \\
   \sin(\theta + \phi) &= \sin \theta \cos \phi + \cos \theta \sin \phi
   \end{align*}
   \]

We’re all probably somewhat rusty. I know I am.
2.2 Rotation

Consider rotating the point \((x, y)\) by \(\theta\) about the origin.

\[
x = r \cos \phi \\
y = r \sin \phi \\
x' = r \cos(\theta + \phi) \\
y' = r \sin(\theta + \phi)
\]

With a little magic:

\[
x' = x \cdot \cos \theta - y \cdot \sin \theta \\
y' = x \cdot \sin \theta + y \cdot \cos \theta
\]

What’s our transformation matrix look like?

2.3 Scaling

1. “Contract” or “expand” a point (polygon).

2. Point moves in relation to origin.

3. Differential, uniform scalings.

\[
x' = s_x \cdot x \\
y' = s_y \cdot y
\]

Matrix representation?

2.4 Translation

Move the point:

\[
x' = x + d_x \\
y' = y + d_y
\]

Matrix representation?
2.5 Homogeneous Coordinates

1. Use allows us to achieve translations via matrix multiplications.

2. Add a third coordinate to a point: \((x, y, W)\).

3. Two sets of homogeneous coordinates represent the same point iff they are multiples of each other.

4. A “homogenized” point.

Our translation:

\[
\begin{bmatrix}
    x' \\
    y' \\
    1
\end{bmatrix} =
\begin{bmatrix}
    1 & 0 & d_x \\
    0 & 1 & d_y \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    x \\
    y \\
    1
\end{bmatrix}
\]

2.6 Composing Transformations

Can we combine transformations?

1. Consider composing two translations: \(d_{x_1}, d_{y_1}\) and \(d_{x_2}, d_{y_2}\).

2. Consider two scalings.

3. Consider two rotations.

2.7 Types of Transformations


2. Affine. Parallelism of lines preserved, but not lengths nor angles.

3. Shear (affine).

Consider the x-shear transformation:

\[
\begin{bmatrix}
    1 & a & 0 \\
    0 & 1 & 0 \\
    0 & 0 & 1
\end{bmatrix}
\]
What’s the y-shear transformation matrix look like?

2.8 General Compositions

1. How do we rotate about an arbitrary point?

2. How do we scale about an arbitrary point?