1 Administrivia

Announcements

Assignment

Read 4.6–9.

From Last Time

Project days.

Outline

1. 2-D transformations: rotation, translation, scaling.

Coming Up

Concatenation of transformations, transformation matrices.
2 2-D Transformations

Three primitive transformation:

1. Rotation.
2. Scaling.
3. Translation.

We’ll consider each in turn.

The idea is to perform all transformations via matrix multiplications:

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} =
\begin{bmatrix}
  \alpha_{1,1} & \alpha_{1,2} \\
  \alpha_{2,1} & \alpha_{2,2}
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]

2.1 Preliminaries

For now, we assume you’re familiar with:

1. Vector spaces and their properties.
2. Dot product.
3. Magnitude of a vector: \(|v| = \sqrt{v \cdot v}.
4. Angle between two vectors:
   \[\theta = \cos^{-1}\left(\frac{v \cdot \omega}{|v||\omega|}\right)\]
5. Properties of matrices.
6. Some trigonometry:
   \[
   \begin{align*}
   \cos(\theta + \phi) &= \cos \theta \cos \phi - \sin \theta \sin \phi \\
   \sin(\theta + \phi) &= \sin \theta \cos \phi + \cos \theta \sin \phi
   \end{align*}
   \]

We’re all probably somewhat rusty. I know I am.
2.2 Rotation

Consider rotating the point \((x, y)\) by \(\theta\) about the origin.

\[
\begin{align*}
x &= r \cos \phi \\
y &= r \sin \phi \\
x' &= r \cos(\theta + \phi) \\
y' &= r \sin(\theta + \phi)
\end{align*}
\]

With a little magic:

\[
\begin{align*}
x' &= x \cdot \cos \theta - y \cdot \sin \theta \\
y' &= x \cdot \sin \theta + y \cdot \cos \theta
\end{align*}
\]

What’s our transformation matrix look like?

2.3 Scaling

1. “Contract” or “expand” a point (polygon).

2. Point moves in relation to origin.

3. Differential, uniform scalings.

\[
\begin{align*}
x' &= s_x \cdot x \\
y' &= s_y \cdot y
\end{align*}
\]

Matrix representation?

2.4 Translation

Move the point:

\[
\begin{align*}
x' &= x + d_x \\
y' &= y + d_y
\end{align*}
\]

Matrix representation?
### 2.5 Homogeneous Coordinates

1. Use allows us to achieve translations via matrix multiplications.

2. Add a third coordinate to a point: \((x, y, W)\).

3. Two sets of homogeneous coordinates represent the same point iff they are multiples of each other.

4. A “homogenized” point.

Our translation:

\[
\begin{pmatrix}
    x' \\
y' \\
1
\end{pmatrix} =
\begin{bmatrix}
    1 & 0 & d_x \\
    0 & 1 & d_y \\
    0 & 0 & 1
\end{bmatrix}
\begin{pmatrix}
x \\
y \\
1
\end{pmatrix}
\]

### 2.6 Composing Transformations

Can we combine transformations?

1. Consider composing two translations: \(d_{x1}, d_{y1}\) and \(d_{x2}, d_{y2}\).

2. Consider two scalings.

3. Consider two rotations.

### 2.7 Types of Transformations


2. Affine. Parallelism of lines preserved, but not lengths nor angles.

3. Shear (affine).

   Consider the x-shear transformation:

   \[
   \begin{bmatrix}
   1 & a & 0 \\
   0 & 1 & 0 \\
   0 & 0 & 1
   \end{bmatrix}
   \]
What’s the y-shear transformation matrix look like?

2.8 General Compositions

1. How do we rotate about an arbitrary point?

2. How do we scale about an arbitrary point?