Entailment of Functional Dependencies

Tom Kelliher, CS 318

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1 Administrivia

Announcements

Return homework Monday. Midterm review Monday. Midterm will cover Chapters 1–7.

Assignment

From Last Time

Introduction to functional dependencies.

Outline

1. Demonstrating entailment.
2. Entailment checking algorithm.
3. Entailment checking example.

Coming Up

Midterm review.

2 Demonstrating Entailment

Using Armstrong's Axioms:

- $\overline{XY} \rightarrow \overline{X}$.
- $\overline{X} \rightarrow \overline{Y}$ entails $\overline{XZ} \rightarrow \overline{YZ}$.
- $\overline{X} \rightarrow \overline{Y}$ and $\overline{Y} \rightarrow \overline{Z}$ entail $\overline{X} \rightarrow \overline{Z}$.

show that

1. $\overline{X} \rightarrow \overline{Y}$ and $\overline{X} \rightarrow \overline{Z}$ entail $\overline{X} \rightarrow \overline{YZ}$. (Union)
2. $\overline{X} \rightarrow \overline{YZ}$ entails $\overline{X} \rightarrow \overline{Y}$ and $\overline{X} \rightarrow \overline{Z}$. (Decomposition)

2.1 So What?

1. Generally, when normalizing, we're interested in knowing if $f \in \mathcal{F}^+$.
2. Armstrong's axioms are *sound* and *complete*.

   Therefore, an algorithm exists to do entailment checking.

3 Entailment Checking Algorithm

1. Suppose we want to see if $\overline{X} \rightarrow \overline{Y} \in \mathcal{F}^+$.
2. We check this by seeing if $\overline{Y}$ is in the attribute closure of $\overline{X}$ with respect to $\mathcal{F}^+$.
   (a) This closure is defined:
   $$\overline{X}^+_{\mathcal{F}} = \{A | \overline{X} \rightarrow A \in \mathcal{F}^+\}$$
   (b) $\overline{Y} \subseteq \overline{X}^+_{\mathcal{F}}$ implies $\overline{X} \rightarrow A \in \mathcal{F}^+$ for each $A \in \overline{Y}$.

   By the union property then, $\overline{X} \rightarrow \overline{Y} \in \mathcal{F}^+$.

(c) Note that $\overline{X} \subseteq \overline{X}^+_{\mathcal{F}}$.

Why?

3. Attribute closure algorithm:

```
closure := X̄
repeat
   old := closure
   if there is an FD Z̄ → V̄ ∈ 𝓕 such that Z̄ ⊆ closure then
      closure := closure ∪ V̄
until old = closure
return closure
```

Correctness. Use induction. (Not the entire proof.)

(a) Basis. `closure` $:= \overline{X}$.

Observe $\overline{X} \rightarrow \overline{X} \in \mathcal{F}^+$. Why?

By decomposition, $\overline{X} \rightarrow A, A \in \overline{X}$.

(b) Inductive step. Assume $\overline{X} \rightarrow$ `closure` $\in \mathcal{F}^+$.

Suppose we have $\overline{Z} \rightarrow \overline{V} \in \mathcal{F}$ where $\overline{Z} \subseteq$ `closure`. *Justify each of the following steps.*

Observe `closure` $\rightarrow \overline{Z} \in \mathcal{F}^+$.

So $\overline{X} \rightarrow \overline{V} \in \mathcal{F}^+$.

Therefore $\overline{X} \rightarrow$ `closure` $\cup \overline{V} \in \mathcal{F}^+$.

4 Entailment Checking Example

Suppose we have $\mathbf{R} = (\overline{R}; \mathcal{F})$, where $\overline{R} = ABCDEFGHIJ$ and $\mathcal{F}$ contains

1. $AB \rightarrow C$

2. $D \rightarrow E$

3. $AE \to G$
4. $GD \to H$
5. $ID \to J$

Check whether or not $\mathcal{F}$ entails $ABD \to GH$ and $ABD \to HJ$.