Question Set 4-5

CS 320

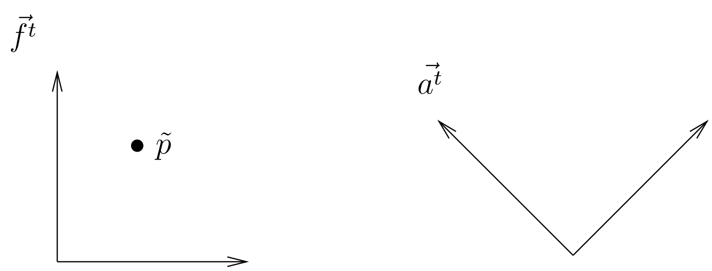
Chapter 4

1. Let

$$S = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   What transformation is accomplished by this matrix?

2. Consider S from the previous problem.



   (a) Describe $\tilde{p} = \vec{\mathbf{f}}^t\mathbf{c} \Rightarrow \vec{\mathbf{f}}^t S\mathbf{c}$ and draw the transformed point.

   (b) Describe $\tilde{p} = \vec{\mathbf{a}}^t\mathbf{c} \Rightarrow \vec{\mathbf{a}}^t S\mathbf{c}$ and draw the transformed point.

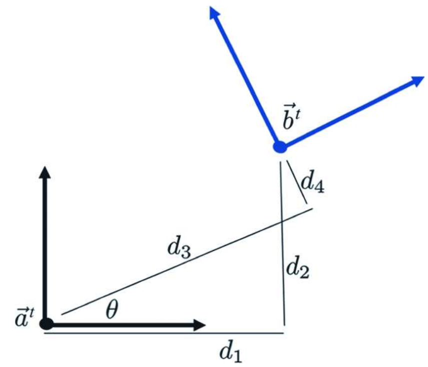
3. Let $\tilde{q} = \vec{\mathbf{f}}^t\mathbf{c}$ and $\vec{\mathbf{a}}^t = \vec{\mathbf{f}}^t A$. What are $\tilde{q}$'s coordinates with respect to $\vec{\mathbf{a}}^t$?

4. State the "left of" rule and illustrate it with an example.

5. Using the definitions of Section 4.2, draw two different sketches illustrating the transformation $\vec{\mathbf{f}}^t \Rightarrow \vec{\mathbf{f}}^t RT$.

6. Suppose $\vec{\mathbf{f}}^t$ is an orthonormal frame, and we apply the transformation $\vec{\mathbf{f}}^t \Rightarrow \vec{\mathbf{f}}^t ST$ where S is a matrix that applies a uniform scale by a factor of 2, and T translates by 1 along the x axis. How far does the frame's origin move, measured in the original units of $\vec{\mathbf{f}}^t$?

7. Consider the following two orthonormal frames $\vec{\mathbf{a}}^t$ and $\vec{\mathbf{b}}^t$



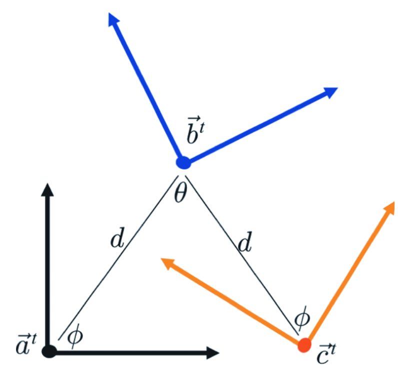
with distances given by the positive quantities d_i.

(a) What are the matrices R and T such that $\vec{\mathbf{b}}^t = \vec{\mathbf{a}}^t TR$?

(b) What are the matrices R and T such that $\vec{\mathbf{b}}^t = \vec{\mathbf{a}}^t RT$?

Your answers should not contain trigonometric terms in the matrix T.

8. Consider the following three frames.



Let $\vec{\mathbf{b}}^t = \vec{\mathbf{a}}^t N$ and $\vec{\mathbf{c}}^t = \vec{\mathbf{a}}^t M$. Express the matrix M in terms of N and θ.

Chapter 5

1. Define:

(a) Object frame

(b) World frame

(c) Eye frame

(d) Object coordinates

(e) World coordinates

(f) Eye coordinates

2. The world frame corresponds neither to any drawn geometry nor to the eye. Yet, it is essential. Why?

3. How do object coordinates differ from eye coordinates?

4. Given

$$\begin{aligned}\vec{\mathbf{o}}^t &= \vec{\mathbf{w}}^t O \\ \vec{\mathbf{e}}^t &= \vec{\mathbf{w}}^t E\end{aligned}$$

   If $\tilde{p} = \vec{\mathbf{o}}^t \mathbf{c}$

   (a) What are $\tilde{p}$'s world coordinates?

   (b) What are $\tilde{p}$'s eye coordinates?

5. How do we move an object?

6. In a right-handed coordinate system, imagine a cube 6 ft. in front of your eyes. $\vec{\mathbf{e}}^t$'s origin is between your eyes, with its x-axis to the right and its y-axis up. $\vec{\mathbf{o}}^t$'s origin is the center of the cube, with its x-axis pointing at $\vec{\mathbf{e}}^t$'s origin and its y-axis to the left.

   (a) For each frame, describe the z-axis.

   (b) The cube is translated 6 ft. along the x-axis with respect to $\vec{\mathbf{o}}^t$. Describe its new position.

   (c) From its original position, the cube is rotated $\pi/2$ radians about the y-axis with respect to $\vec{\mathbf{e}}^t$. Describe its new position.

7. Watch the *Respect the Frame* video, available from the course web site. What frame is used in Version 1, resulting in incorrect behavior? What frame is used in Version 2, resulting in correct behavior?

8. Show how to apply a transformation M to $\vec{\mathbf{o}}^t$ with respect to $\vec{\mathbf{a}}^t$.

9. Explain how $\vec{\mathbf{a}}^t = \vec{\mathbf{w}}^t (O)_T (E)_R$ gives us an auxiliary basis that has $\vec{\mathbf{o}}^t$'s origin and $\vec{\mathbf{e}}^t$'s orientation. How do we construct $(O)_T$ and $(E)_R$?

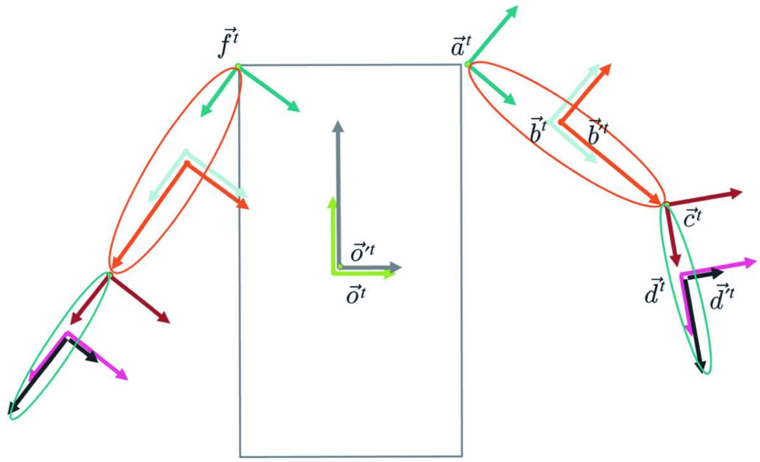
10. Given $\vec{\mathbf{e}}^t = \vec{\mathbf{w}}^t E$ with E such that the eye is located at $[0, 0, 5]$, pointed at $[0, 0, 0]$ (the origin of $\vec{\mathbf{w}}^t$), with an up vector of $[0, 1, 0]$. Let R be a rotation transformation of $\pi/2$ radians about the y-axis. Let $\vec{\mathbf{a}}^t = \vec{\mathbf{w}}^t A$ where $A = (W)_T (E)_R$. ($W = I$; I is the identity matrix.)

    Describe the effect of performing the updates

    (a) $E \leftarrow ER$ (corresponding to the transformation $\vec{\mathbf{e}}^t = \vec{\mathbf{w}}^t E \Rightarrow \vec{\mathbf{w}}^t ER$)

    (b) $E \leftarrow ARA^{-1}E$ (corresponding to the transformation $\vec{\mathbf{e}}^t = \vec{\mathbf{w}}^t E \Rightarrow \vec{\mathbf{w}}^t ARA^{-1}E$)

11. Consider this robot:



where

(a) $\vec{\mathbf{o}}^t = \vec{\mathbf{w}}^t O$ (torso frame)

(b) $\vec{\mathbf{a}}^t = \vec{\mathbf{o}}^t A$ (right shoulder joint frame)

(c) $\vec{\mathbf{b}}^t = \vec{\mathbf{a}}^t B$ (right upper-arm frame)

(d) $\vec{\mathbf{c}}^t = \vec{\mathbf{a}}^t C$ (right elbow joint frame)

What is the advantage of expressing $\vec{\mathbf{a}}^t$ with respect to $\vec{\mathbf{o}}^t$, rather than respect to $\vec{\mathbf{w}}^t$?

12. Suppose that we have a scene with a jet airplane flying in the sky. Suppose that the jet's geometry is described with respect to the jet's own frame, $\vec{\mathbf{j}}^t = \vec{\mathbf{w}}^t J$. Let this frame have its origin in the cockpit with the negative z-axis pointing out the front window. Suppose we wish to render the scene from the point of view of the pilot. Given a point on some other object, $\tilde{p} = \vec{\mathbf{o}}^t \mathbf{c}$, what coordinate vector should we pass to the renderer to draw this point?

13. Consider the robot of question 11. Suppose that we want to rotate the right shoulder joint with respect to a frame whose origin is at the center of the joint but has axes that align with those of the eye. How would this be implemented?