Question Set 2-3

CS 320

Chapter 2

1. Identify the types (basis, coordinate vector, matrix, point, vector) of the variables in this equation
$$\vec{v} = \vec{\mathbf{b}}^t M^{-1}\mathbf{c}$$

2. Draw a figure corresponding to
$$\vec{\mathbf{b}}^t\mathbf{c} \Rightarrow \vec{\mathbf{b}}^t M\mathbf{c}$$
and express this mathematical statement in words.

3. Draw a figure corresponding to
$$\vec{\mathbf{b}}^t \Rightarrow \vec{\mathbf{b}}^t M$$
and express this mathematical statement in words.

4. If
$$\vec{v} \cdot \vec{w} = 0$$
then what do we know?

5. Define the term *3D orthonormal basis*.

6. How is the vector
$$\vec{v} \times \vec{w}$$
related to the two vectors in the expression?

7. Which of the following are valid expressions in our notation and, if valid, what is the resulting type (invalid, basis, coordinate vector, matrix, point, vector)
   (a) $\vec{\mathbf{b}}^t M$
   (b) $\mathbf{c}M$
   (c) $M^{-1}\mathbf{c}$
   (d) $\vec{\mathbf{b}}^t N M^{-1}\mathbf{c}$

8. Given that $\vec{\mathbf{a}}^t = \vec{\mathbf{b}}^t M$, what are the coordinates of the vector $\vec{\mathbf{b}}^t N\mathbf{c}$ with respect to the basis $\vec{\mathbf{a}}^t$? (Your answer will be a mathematical expression.)

9. Given that the transformation $\mathcal{T}(\vec{v})$ is defined as $\mathcal{T}(\vec{v}) = \vec{v} + \vec{k}$, show that $\mathcal{T}(\vec{v})$ is not a linear transformation.

Chapter 3

1. What does $\tilde{o}$ represent?
2. Practically speaking, what is the difference between a basis and a frame?
3. Given points $\tilde{p}$ and $\tilde{q}$, how do we construct a vector from $\tilde{p}$ to $\tilde{q}$?
4. Is translation linear or affine?
5. Is rotation linear or affine?
6. Does $[3, 7, 8, 1]^t$ represent a point or a vector?
7. Express the affine matrix
$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
as the product of its translational and linear factors.
8. Which of these transformations are RBTs: rotation, scale, translation?
9. These two operations make sense: $\tilde{p} - \tilde{q} = \vec{v}$ (point-point subtraction), and $\tilde{q} + \vec{v} = \tilde{p}$ (point-vector addition). On its face, this operation does not make sense (What is the product of a scalar and a point? What is the sum of two such objects?):
$$\alpha\tilde{p_1} + (1 - \alpha)\tilde{p_2},$$
in which α is on the interval $[0, 1]$. Algebraically manipulate this operation into a form that does make sense. What is the type (vector or point) of this operation? What is its geometric interpretation?