Question Set 6

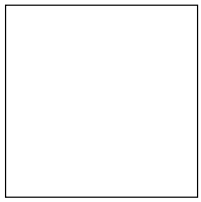
CS 320

Chapter 6

1. The linear interpolation (LERP) of two rotation matrices,

$$R_\alpha := (R_1 R_0^{-1})^\alpha R_0,$$

when applied to a frame, causes the frame's basis vectors to move along a straight line. This implies that points associated with the frame move along a straight line as well. Consider LERPing two rotation matrices to rotate this square by 90°:



In a single hand-drawn figure, show the result of the LERP operation on this square for $\alpha = 0$ and $\alpha = 0.5$. Explain why this is not rotation.

2. Define axis/angle rotation and explain how a quaternion encodes this rotation mechanism.

3. Show how to construct a quaternion representing a rotation of θ about $\hat{\mathbf{k}}$. Do not assume that $\hat{\mathbf{k}}$ is of unit length.

4. What is the multiplicative inverse of the quaternion you constructed in the previous question? Give a geometric reason as to why this is the multiplicative inverse.

5. Assuming that $\hat{\mathbf{k}}$ is of unit length, show that any quaternion constructed from it is also of unit length.

6. Does the coordinate four-vector $[\hat{\mathbf{c}}, 1]^t$ represent a point or a vector?

7. Write the rotation by $\frac{2\pi}{3}$ about $[3, 1, 5]$ as a unit norm quaternion. Show work.

8. Extract the angle and axis from the quaternion $[0.8660, 0, 0.2236, 0.4472]$. Assume that the angle is measured in radians. Show work.