Binary Addition and Subtraction

Tom Kelliher, CS 220

Oct. 3, 2003

1 Administrivia

Announcements

Written assignment due Monday.

Assignment

Read 3.10

From Last Time

Decoders, encoders, muxes.

Outline

1. Binary Adders
2. Unsigned binary subtraction; complements.

Coming Up

Combined signed addition/subtraction.

2 Binary Adders

1. Two half-adders and an OR gate give us a full binary adder. In the text, note how an earlier computed XOR is used to eliminate a gate from the carry equation.

2. Full binary adder: three inputs, two outputs.

3. Ripple carry adder: example of reuse and divide and conquer.

   (a) Wire together n full binary adders in order to add two n bit numbers.

   Wiring example.

   (b) Running time of a ripple carry adder.

   Running time of a full binary adder is O(1). Ripple carry adder?

   Excessive!!!

4. An O(1) (!!!) adder.

   (a) Important equations (briefly explain):

   Carry generate at bit position i: $G_i = A_i B_i$.

   Carry propagate at position i: $P_i = A_i \oplus B_i$.

   (b) Carry-in is C_0.

   $C_1 = G_0 + P_0 C_0$.

   $C_2 = G_1 + P_1 G_0 + P_1 P_0 C_0$.

   $C_3 = G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0$.

   Etc.

   (c) What's the circuit depth of C_i?

   (d) What's wrong with this picture?

5. Carry lookahead addition.

(a) A different type of divide and conquer adder using a tree hierarchy to compute and distribute carry information.

(b) What's the height of a binary tree?

(CLA is not binary, more like quad, but that's ok.)

(c) What's the running time of a CLA?

3 Unsigned Binary Subtraction

Unsigned here means we can use a minus sign. Realistic?

Let A = 110101 and B = 011010. Compute $A - B$ and $B - A$.

$A - B$: fine. $B - A$: borrow out of msb.

1. Actual value computed: $2^n + B - A$.
2. We want $-(A - B)$.
3. So, compute $2^n - (2^n + B - A) = A - B$.

The borrow into the msb leads us to the notion of complements.

3.1 Complements

Used for *signed* representations.

1. Diminished radix complement: 1's complement.

   (a) The 1's complement of an n bit binary number A is $2^n - 1 - A$.

   (b) What's the bit representation of $2^n - 1$? The one's complement of A? A plus its one's complement?

2. Radix complement: 2's complement.

   (a) The 2's complement of an n bit binary number A is $2^n - A$.

   (b) 1's complement plus one.

       Two's complement of A? A plus its two's complement?

Adding to subtract:

1. Denote the 2's complement of B as B'.

   Recall $B' = 2^n - B$.

2. $A - B = A + B' - 2^n$.

   Note we should get a carry out of the msb when we perform $A + B'$.

3. Work the two examples again.