CS250 Lab 11 – Context-Free Pumping Lemma

Objectives: In this lab you will learn how to

- prove languages are not context-free

Recall the definition of the context-free pumping lemma:

Let L be an infinite context-free language. Then there exists some positive integer m such that any w that is a member of L with $|w| \geq m$ can be decomposed as $w = uvxyz$, with $|vxy| \leq m$, and $|vy| \geq 1$, such that $w_i = uv^ixy^iz$, is also in L for all $i = 0, 1, 2, ...$

In other words, any sufficiently long string in L can be broken down into five parts such that any number of repetitions of the 2^{nd} and 4^{th} parts will still yield in a string in L.

JFLAP treats the context-free pumping lemma as a two-player game. One player, player A, is trying to prove that the language is not context-free, and the other player, player B, is trying to make it as hard as possible for player A to do so. The game is played like this:

1. Player B picks an integer for m.
2. Player A picks a string w such that w is a member of L and $|w| \geq m$.
3. Player B picks the partition of w into $uvxyz$ such that $|vxy| \leq m$ and $|vy| \geq 1$.
4. Player A picks an integer i such that uv^ixy^iz is not a member of L. If player A can do so player A wins, otherwise, player B wins.

If player A can pick a strategy such that they will always win regardless of player B's choices, it is equivalent to proof that the language is not context-free. JFLAP takes the role of player A, and you take the role of player B, with a few examples that are included.

Assignment 1:
Explain in your own words why the existence of a strategy for player A that always wins is equivalent to proof that the language is not context-free.

Assignment 2:
Play the game for a number of the examples. There is at least one context-free language there. Identify and explain how you won the game for the context-free languages. Does winning the game mean the language IS context-free? Why or why not?

Assignment 3:
Write out formal pumping lemma proofs for exercises 7d, g on p221.

Submit your files in goucherLearn for grading.